On Wireless Board-to-Board Communication with Cascaded Butler Matrices

Johannes Israel, Andreas Fischer
Institute of Numerical Mathematics
SFB 912 HAEC
Technische Universität Dresden
01062 Dresden, Germany
Email: {Johannes.Israel, Andreas.Fischer}@tu-dresden.de

Meik Dörpinghaus
Vodafone Chair Mobile Communications Systems
SFB 912 HAEC
Technische Universität Dresden
01062 Dresden, Germany
Email: Meik.Doerpinghaus@tu-dresden.de

***Abstract*—Antenna arrays fed by cascaded Butler matrix networks can be used to enable a very flexible wireless communication network between computer boards. However, the quality of the wireless links, particularly considering interference, depends on the design of the Butler matrices and on certain topology parameters, like the distance of boards, placement of antenna arrays, and the distance of antenna elements within the antenna arrays. In this paper, we model such a wireless multi-link board-to-board communication scenario and investigate the influence of these parameters. The Worst-Case Signal-to-Interference-and-Noise-Ratio (WCSINR) of a link is used as a measure of quality. We show that an optimization of the topological parameters significantly improves the (average) WCSINRs and yields a better performance of the communication links.**

I. Introduction

The investigation of new concepts for wireless interconnections of chips in a compute system to enable flexibility together with energy efficiency is one of the key aspects within the Collaborative Research Center 912 "HAEC" [1]. Several chips are placed on parallel computer boards in a regular grid structure. Each chip is equipped with antenna arrays allowing for wireless communication with the chips on the neighboring board. The application of beamforming techniques [2] for wireless board-to-board communication would enable high adaptivity and focused beams. The use of finite resolution devices (phase shifters, amplifiers) for analog receive beamforming in the context of board-to-board communication has been investigated in [3], [4], and [5]. On the one hand, the design of active circuit components for the application of beamforming techniques in the intended high frequency domain is challenging. On the other hand, the adjustment of optimal beamforming weights might cause too much latency. Therefore, a beam *switching* architecture based on Butler matrices [6] for the wireless chip-to-chip communication is considered. The Butler matrix is a switching network that consists of passive circuit elements only. A cascade of several Butler matrices can be used to generate spatial beams. The number of different beampatterns provided by such a cascaded Butler matrix switching network is equal to the number of elements in the antenna array. The resulting beampatterns would generally not be adequate for all wireless links that need to be served, possibly at the same time. Hence, it is of special interest to optimize certain parameters such that a sufficient link quality is reached. These parameters might be

(a) the distance d_B of the parallel computer boards,
(b) the antenna element spacing d_A,
(c) the placement of antenna arrays on top of the chips,
(d) the position of chips on the parallel boards,
(e) the design of the cascaded Butler matrix network.

In this paper, we contribute a system model for multi-link wireless board-to-board communication based on cascaded Butler matrices and investigate the influence of the parameters (a)–(c). Their optimal configuration depends, of course, on the geometrical conditions given by the concrete application.

The placement of chips for board-to-board communication with cascaded 4×4 Butler matrices has been investigated in [7]. There, it is shown that a certain checkerboard positioning rule maximizes the number of shortest links with highest array response. Here, we assume that the chips are placed in a fixed regular grid structure with equal positions on the parallel boards, which reduces the required space. Thus, option (d) is not considered. The Butler matrix design (e) is kept fixed as well.

The paper is structured as follows. In Section II, the Butler matrix and the cascaded switching network is described and characterized. The system model and the WCSINR is introduced in Section III. In Section IV, simulation results are provided and it is shown that the adjustment of the parameters in (a)–(c) has a significant impact on the achievable quality of the wireless links. Conclusions are given in Section V.

II. Cascaded Butler matrix

The $M \times M$ Butler matrix is a passive switching network for a linear antenna array with M antenna elements and consists of fixed phase shifters and passive four-port hybrid power dividers. The m-th output port is connected to the m-th antenna element and the input ports are the interfaces between the Butler matrix and the RF front end. A mathematical description of a general $M \times M$ Butler matrix is given by $\mathbf{B} = [\mathbf{b}_1, \ldots, \mathbf{b}_M]$ with

$$b_{mk} := \frac{1}{M} e^{\frac{j\pi}{M}(2mk-m-k+1)} \quad \text{for } m, k \in \{1, \ldots, M\}. \quad (1)$$

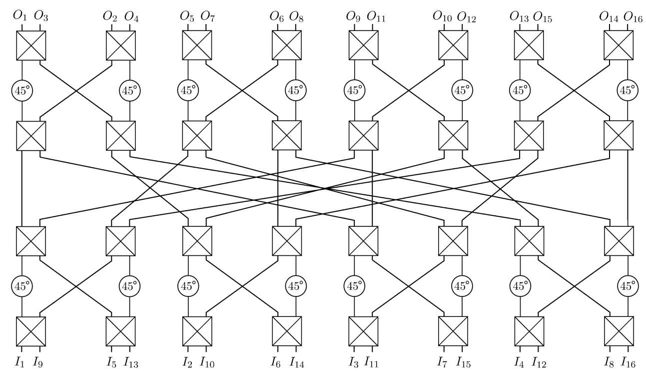


Figure 1. Schematic of a cascade of eight 4×4 Butler matrices

Choosing the k-th input port of the Butler matrix corresponds to the beamforming vector $\mathbf{b}_k$, i.e., the k-th column of $\mathbf{B}$, and yields a certain beampattern. For an incoming signal from direction θ the array response is given by $R_k(\theta) := |\mathbf{b}_k^{\mathsf{H}}\mathbf{a}(\theta)|^2$, where $\mathbf{a}(\theta)$ is the array steering vector with

$$a_m = e^{j\frac{2\pi}{\lambda}d_A(m-1)\sin\theta} \quad \text{for } m \in \{1,\ldots,M\}$$

and λ denotes the signal's wavelength. In the rest of this section we assume that the antenna element distance is $d_A = 0.5\lambda$. The highest possible array response is $R_k(\theta^\star) = 1$ and is achieved at the main response axes (MRA) which are given by

$$\theta_k^\star := \begin{cases} \arcsin\left(\frac{2k-1}{M}\right) & \text{for } k = 1,\ldots,\frac{M}{2}, \\ \arcsin\left(\frac{2k-1}{M} - 2\right) & \text{for } k = \frac{M}{2}+1,\ldots,M, \end{cases} \tag{2}$$

see [7] for details. Note that there are other alternatives for the formal representation of a Butler matrix [7], [8] which yield the same MRA. The relevant property of a Butler matrix is the constant relative phase distance of $(2k-1)\pi/M$ between the components of the beamformer $\mathbf{b}_k$ for $k \in \{1,\ldots,M\}$.

In order to feed an $M \times M$ *quadratic* antenna array, we use a cascade of $2M$ Butler matrices as exemplarily shown in Figure 1 for $M = 4$. Such a cascade can be simply described by the Kronecker product

$$\mathbf{C} := \mathbf{B} \otimes \mathbf{B} = \begin{pmatrix} b_{11}\mathbf{B} & \ldots & b_{1M}\mathbf{B} \\ \vdots & \ddots & \vdots \\ b_{M1}\mathbf{B} & \ldots & b_{MM}\mathbf{B} \end{pmatrix} \in \mathbb{C}^{M^2 \times M^2}. \tag{3}$$

To determine the MRAs for the beampatterns provided by the cascaded Butler matrix $\mathbf{C}$ we introduce for $m, n \in \{1,\ldots,M\}$

$$f_m := \begin{cases} \frac{2m-1}{M} & \text{for } m \le \frac{M}{2}, \\ \frac{2m-1}{M} - 2 & \text{else,} \end{cases} \tag{4}$$

and

$$g_n := \begin{cases} \frac{2n-1}{M} & \text{for } n \le \frac{M}{2}, \\ \frac{2n-1}{M} - 2 & \text{else.} \end{cases} \tag{5}$$

By $\mathbf{c}_m^n$, we denote the $((m-1)M+n)$-th column of $\mathbf{C}$, i.e.,

$$\mathbf{C} = [\mathbf{c}_1^1,\ldots,\mathbf{c}_1^M,\mathbf{c}_2^1,\ldots,\mathbf{c}_2^M,\ldots,\mathbf{c}_M^1,\ldots,\mathbf{c}_M^M].$$

The array response for the beamformer $\mathbf{c}_m^n$ depends on the polar angle θ and the azimuthal angle φ and is calculated as $R_{m,n}(\theta,\varphi) = |(\mathbf{c}_m^n)^{\mathsf{H}}\mathbf{a}(\theta,\varphi)|^2$ with the steering vector $\mathbf{a}(\theta,\varphi)$ defined by

$$a_m := e^{j\frac{2\pi}{\lambda}d_A\left(\sin\theta\cos\varphi\cdot((m-1) \bmod M) + \sin\theta\sin\varphi\cdot\left\lfloor\frac{m-1}{M}\right\rfloor\right)}$$

for $m \in \{1,\ldots,M^2\}$. It can be shown that the maximal array response $R_{m,n}(\theta^\star,\varphi^\star) = 1$ is achieved if and only if

$$f_m^2 + g_n^2 \le 1 \tag{6}$$

holds, see [7]. In that case the MRA is given by

$$\theta_{m,n}^\star := \operatorname{sgn}(f_m)\cdot\arcsin\left(\sqrt{f_m^2+g_n^2}\right), \tag{7}$$

$$\varphi_{m,n}^\star := \arctan\left(\frac{g_n}{f_m}\right). \tag{8}$$

Because of condition (6), only a subset of the M^2 input ports of a cascaded Butler matrix yields a beampattern with maximal array response $R_{m,n}(\theta^\star,\varphi^\star) = 1$ at the MRA. For an 8×8 Butler matrix, 52 out of the 64 input ports satisfy (6).

III. Board-to-board communication with cascaded Butler matrices

At the beginning, we define some general conditions for a board-to-board communication scenario that are used throughout the remainder of this paper. Each of the two parallel boards is equipped with 16 chips arranged in a regular 4×4 grid. The distance between the centers of two neighboring chips on the same board is 2.5 cm and each board is quadratic with a size of 10 cm × 10 cm. The quadratic antenna arrays have 8×8 elements and are fed by cascaded 8×8 Butler matrices as described in Section II.

In a first setting, we assume that each chip is equipped with only a single antenna array. Moreover, the antenna arrays have the same positions on all chips of the parallel boards, i.e., we consider a "face-to-face" positioning of the arrays, with Tx arrays on the upper board and Rx arrays on the lower board. This simple setting is called Type I placement, a sketch is shown in Figure 2. For the highlighted Rx antenna array (on the lower board) it is also shown, where the main beams of the 52 beampatterns (with array response 1) "hit" the plane with the Tx antenna arrays on the upper board in case of $d_B = 2$ cm. For ease of notation, the board distance d_B is understood as the distance between the Tx and Rx antenna planes. A possible way to achieve bidirectional communication is the use of time-division multiplexing.

In principle, each chip shall be able to communicate with every chip on the neighboring board. Thus, 256 possible chip-to-chip links have to be taken into account. Two properties are of special interest: there should be appropriate beams for all possible links, while the side lobes in the corresponding beampatterns should yield only slight interferences. We assume that interference caused by reflections is negligible due to path and reflection attenuation. To determine the Signal-to-Interference-and-Noise Ratio (SINR) for a single desired

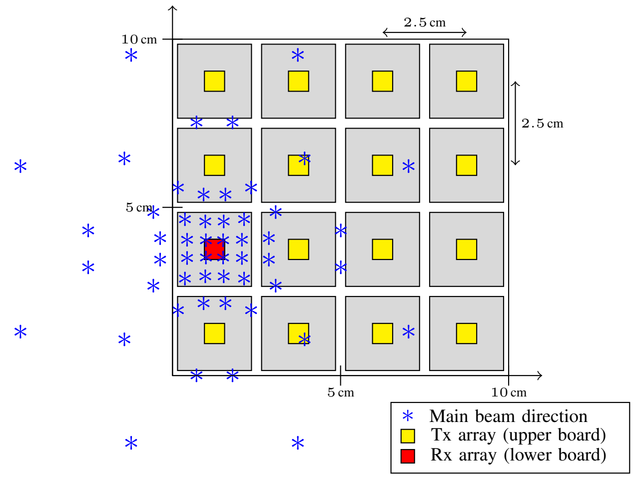


Figure 2. Type I placement of arrays on neighboring boards

link from transmitter q to receiver r we consider the array response for this link and the interferences that are caused by all 15 possible interfering sources at the desired receiver r. Then, for a given noise power σ_n^2, the SINR depends on q and r and is given by

$$\mathrm{SINR}_{q,r} = 10\log\left(\frac{P_q\left(|\mathbf{w}_r^{\mathsf{H}}\mathbf{a}_{r,q}|\cdot|\mathbf{w}_{t,q}^{\mathsf{H}}\mathbf{a}_{t,q}|\right)^2}{\sum\limits_{\substack{l=1\\l\neq q}}^{16} P_l\left(\frac{d_q}{d_l}\cdot|\mathbf{w}_r^{\mathsf{H}}\mathbf{a}_{r,l}|\cdot|\mathbf{w}_{t,l}^{\mathsf{H}}\mathbf{a}_{t,l}|\right)^2+\sigma_n^2}\right). \tag{9}$$

For $l \in \{1,\ldots,16\}$, P_l is the transmit power of the l-th transmitter, d_l is the distance of the l-th transmitter to the desired receiver, $\mathbf{w}_r$ is the optimal beamforming vector for the desired link at the receiver (among the set of beamformers provided by the Butler matrix), $\mathbf{w}_{t,l}$ is the beamforming vector chosen at the l-th transmitter, and $\mathbf{a}_{r,l}$ and $\mathbf{a}_{t,l}$ are the given steering vectors at the receiver and the transmitters. For a given desired link the parameters $d_l, \mathbf{a}_{r,l}, \mathbf{a}_{t,l}$ and also $\mathbf{w}_r$ are well defined by means of the topology. Formula (9) can be derived for example from [9, Chapter 7.2] under certain assumptions on d_l, the transmission bandwidth, and the dimension of the antenna arrays.

In order to evaluate the influence of certain system parameters (described in (a)–(c) in Section I) we define a worst-case scenario. In this scenario all transmitters are active, radiating the same signal power, i.e., $P_1 = P_2 = \ldots = P_{16}$. Moreover, each transmitter $l \in \{1,\ldots,16\}$ chooses the beamformer $\mathbf{w}_{t,l}$ that yields the highest array response in the direction of the desired receiver r, i.e., maximal interference. The SINR value that is obtained under these assumptions is called the worst-case SINR (WCSINR) of this link. For the setting shown in Figure 2, we provide some numerical data for all 16 possible desired links to the highlighted desired receiver r in Table I. Therein, the two subtables contain the array response at the receiver and the WCSINR values for this setting at (exemplarily) 15 dB SNR, i.e., $10\log_{10}(P_q/\sigma_n^2) = 15$. The entry in the k-th row and l-th column corresponds to the

Table I
ARRAY RESPONSE AND WCSINR FOR THE SETTING SHOWN IN FIGURE 2, DESIRED RECEIVER r IS HIGHLIGHTED, $d_B = 2$ cm, $d_A = 0.5\lambda$, PLACEMENT TYPE I

Array responses $\lvert\mathbf{w}_r^{\mathsf{H}}\mathbf{a}_{r,q}\rvert^2$				WCSINR (dB) at 15 dB SNR			
0.35	0.85	0.72	0.56	-3.1	9.4	-8.0	0.0
0.25	0.99	0.85	0.70	-1.4	6.8	2.7	-5.0
0.17	0.25	0.35	0.26	-0.8	-2.4	-4.0	-10.8
0.25	0.99	0.85	0.70	1.7	14.0	3.0	-4.8

case, where the desired transmitter q is located at the chip in the k-th row and l-th column of the 4×4 chip grid for $k, l \in \{1,2,3,4\}$. For example, if the desired transmitter is on the chip at the bottom left corner, the WCSINR of the link between this transmitter and the highlighted desired receiver is 1.7 dB.

From the results in Table I, we observe that the array response is lowest for such links, where the desired transmitter is in the same row or in the same column as the (highlighted) receiver of the chip grid, see third row and first column in the subtable. The beams provided by the cascaded Butler matrix for these links would not be suitable. Indeed, the lowest possible array response occurs in the case, where chips are located face to face, i.e., for links with the shortest possible distance. We emphasize that those Butler matrix beams, whose MRA have the smallest polar angle θ, are quite narrow and therefore also not appropriate for these face-to-face links. Owing to the Butler matrix design, there is no beam pointing to the array perpendicular and also no main beam with an azimuthal angle being an integer multiple of 90°. The links with lowest array response are, however, not necessarily the links with the lowest WCSINR, see Table I. In summary, it might be beneficial to optimize parameters of the overall setting to improve the WCSINR values.

IV. SIMULATIONS AND IMPROVEMENT OF PARAMETERS

As discussed in the previous section the Type I placement (face-to-face positioning) seems to be improvable. In HAEC [10], each chip shall be equipped with four quadratic antenna arrays – one in each polarization domain for Tx and Rx, respectively. Four feasible positions for the four types of arrays imply 24 possibilities for their placement on top of each chip. Taking into account all 32 chips on two neighboring boards yields a total number of $24^{32} \approx 1.5\cdot10^{44}$ different array placements. Even if many of these options can be discarded due to symmetry properties, an optimization with respect to the placement of arrays might be quite challenging. Instead of that, we suggest a homogeneous allocation of the four arrays on all chips on a board such that transmitter and receiver for the same polarization domain are located on opposite corners, see Figure 3. The same placement of arrays on the opposite board implies that no link with an azimuthal angle of $0°, 90°, 180°$, or $270°$ exists. This is desirable according to the results in Table I in the previous section. We refer to this setting as Type II placement. As it is also shown in Figure 3, we assume an equidistant placement of the antenna arrays, i.e., a constant distance of 1.25 cm between the centers of neighboring arrays.

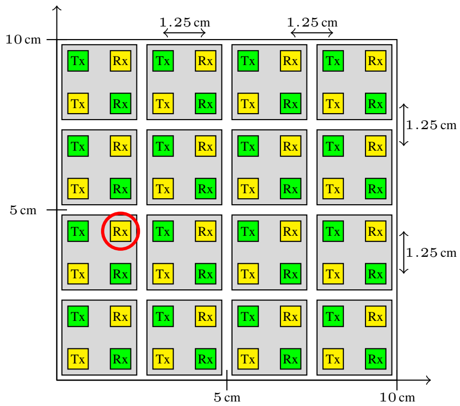


Figure 3. Type II placement of chips and arrays on a board

In Table II, we provide the array response and WCSINR values at 15 dB SNR in the same way as in Table I, but for the Type II placement of antenna arrays. The desired receive antenna array r is marked by a red circle in Figure 3. We assume that there is no crosstalk between the different polarization domains.

Table II
ARRAY RESPONSE AND WCSINR FOR THE SETTING SHOWN IN FIGURE 3, DESIRED RECEIVER r IS HIGHLIGHTED, $d_B = 2\,\text{cm}$, $d_A = 0.5\lambda$, PLACEMENT TYPE II

Array responses $|\mathbf{w}_r^{\mathrm{H}}\mathbf{a}_{r,q}|^2$

0.60	0.60	0.87	0.39
0.39	0.39	0.60	0.67
0.39	0.39	0.60	0.67
0.60	0.60	0.87	0.39

WCSINR (dB) at 15 dB SNR

6.3	6.3	7.3	-9.4
5.8	5.7	3.6	0.3
5.8	5.7	3.6	0.3
6.3	6.3	7.3	-9.4

In comparison to the results in Table I we observe that the array response and WCSINR of some links are degraded. On the other hand, the average response and WCSINR level increases and we obtain way more homogenous results.

The board distance is a parameter that influences strongly the angles of the possible links. Table III contains results for the same setting as in Table II but for a larger board distance of $d_B = 8\,\text{cm}$. Obviously, the increased board distance yields much better results: all possible links to the highlighted receive node provide a WCSINR of at least 9.6 dB.

Table III
ARRAY RESPONSE AND WCSINR FOR THE SETTING SHOWN IN FIGURE 3, DESIRED RECEIVER r IS HIGHLIGHTED, $d_B = 8\,\text{cm}$, $d_A = 0.5\lambda$, PLACEMENT TYPE II

Array responses $|\mathbf{w}_r^{\mathrm{H}}\mathbf{a}_{r,q}|^2$

0.89	0.89	0.97	0.85
0.92	0.92	0.89	0.99
0.92	0.92	0.89	0.99
0.89	0.89	0.97	0.85

WCSINR (dB) at 15 dB SNR

11.0	10.6	11.0	12.4
10.4	9.8	11.0	9.6
10.4	9.8	11.0	9.6
11.0	10.6	11.0	12.4

In the rest of this section, we want to determine optimized parameter settings for several scenarios based on the following constrained parameters:

- antenna element spacing: $0.25\lambda \leq d_A \leq 0.75\lambda$
- board distance: $2\,\text{cm} \leq d_B \leq 10\,\text{cm}$
- placement of arrays: Type I or Type II

Table IV
OPTIMIZED PARAMETERS W.R.T. AVERAGE WCSINR AT 15 dB SNR

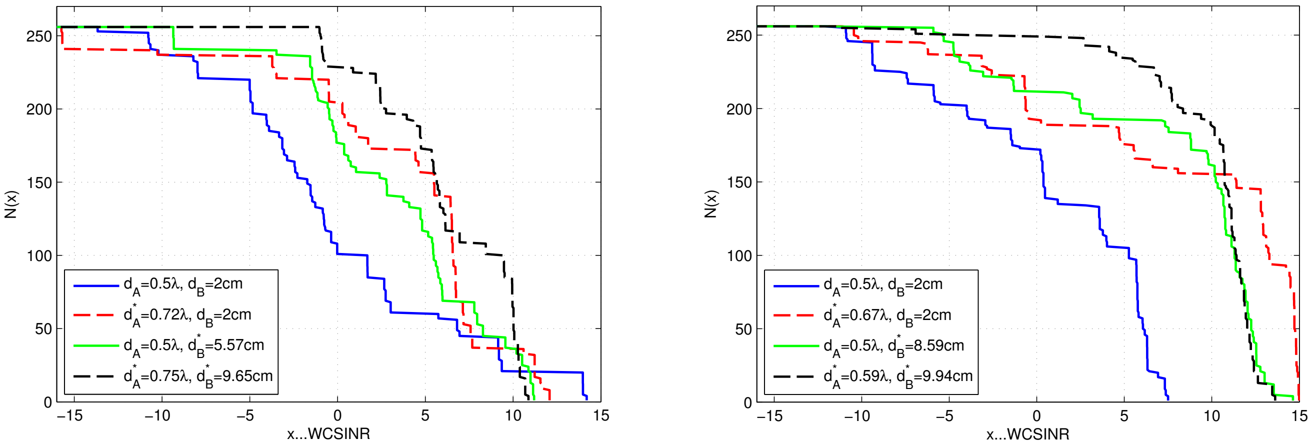
parameters fixed	parameters optimized	array response averaged	WCSINR averaged
$d_B = 2\,\text{cm}$ $d_A = 0.5\lambda$ Type I placement	-	0.57	0.18 dB
$d_B = 2\,\text{cm}$ $d_A = 0.5\lambda$ Type II placement	-	0.59	0.93 dB
$d_B = 2\,\text{cm}$ Type I placement	$d_A^\star = 0.72\lambda$	0.54	3.72 dB
$d_B = 2\,\text{cm}$ Type II placement	$d_A^\star = 0.67\lambda$	0.78	8.28 dB
$d_A = 0.5\lambda$ Type I placement	$d_B^\star = 5.57\,\text{cm}$	0.55	3.42 dB
$d_A = 0.5\lambda$ Type II placement	$d_B^\star = 8.59\,\text{cm}$	0.87	7.91 dB
Type I placement	$d_A^\star = 0.75\lambda$ $d_B^\star = 9.65\,\text{cm}$	0.63	6.29 dB
Type II placement	$d_A^\star = 0.59\lambda$ $d_B^\star = 9.94\,\text{cm}$	0.88	9.85 dB

We emphasize that the choice of the desired receiver node r influences the results for the response level and the WCSINR. Therefore, we use the *average* WCSINR among all 256 possible desired links as objective function, i.e., we determine the WCSINR (in dB) for all desired links from nodes q to r with $q, r \in \{1, \ldots, 16\}$ and calculate the average. The corresponding nonconvex optimization problems can be easily formulated, but are quite complex and difficult to solve. Since only three bounded variables (d_A, d_B and the placement type) are taken into account, we obtain a reasonable approximation of the optimal setting by simulations. Based on this, Table IV shows the average response and average WCSINR at 15 dB SNR for all 256 possible desired links.

In the first four rows in Table IV the board distance is fixed to $d_B = 2\,\text{cm}$. In that case, the polar angles of most of the links are quite large. For the fixed antenna distance $d_A = 0.5\lambda$ an average WCSINR of less than 1 dB is achieved for both placement types.

Allowing for variation in d_A yields improved results. The Type II placement (8.28 dB average WCSINR) clearly outperforms the Type I placement (3.72 dB average WCSINR). We emphasize that $d_A^\star > 0.5\lambda$ holds for all results. In comparison to the standard case $d_A = 0.5\lambda$, an increased antenna distance implies on the one hand decreased polar angles of the MRA and smaller main beams but, on the other hand, also yields that grating lobes occur in the radiation patterns causing additional interference. However, we observed that these grating lobes are actually used for some links, i.e., the highest array response is achieved for a beampattern of the Butler matrix, where the main beam does not point to the desired direction, but where a grating lobe fits to the desired link.

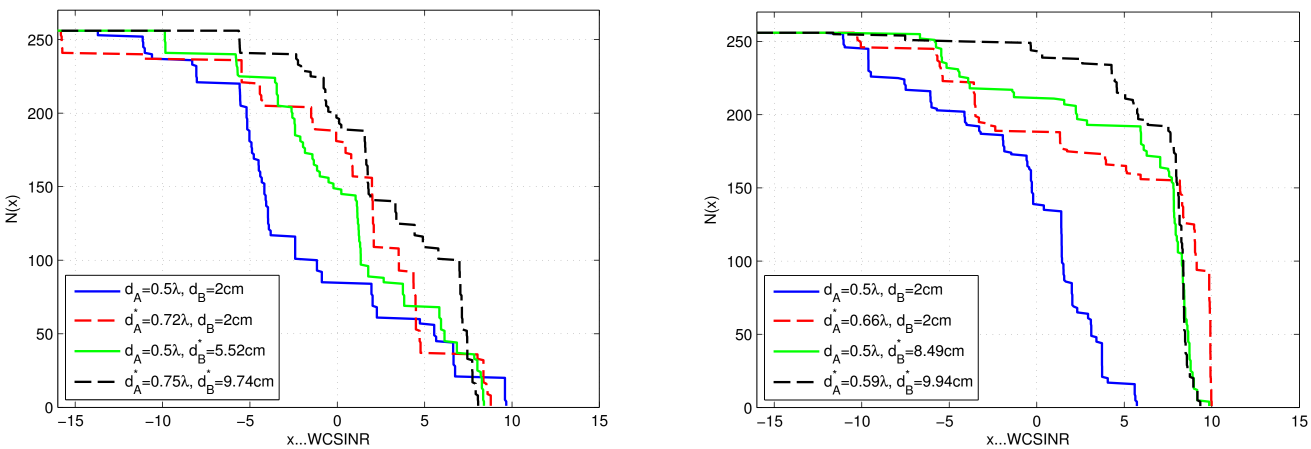
If, instead of d_A, the board distance is variable, the resulting values for the average response and WCSINR are quite similar to the aforementioned case. The Type II placement yields a significantly larger best board distance ($d_B^\star = 8.59\,\text{cm}$) compared to the Type I placement ($d_B^\star = 5.57\,\text{cm}$).



(a) Type I

(b) Type II

Figure 4. $N(x)$ for Type I placement and Type II placement at 15 dB SNR



(a) Type I

(b) Type II

Figure 5. $N(x)$ for Type I placement and Type II placement at 10 dB SNR

If the board distance d_B as well as the antenna distance d_A are variable, the best board distance is almost 10 cm for both placement types. By means of the Type II placement an average WCSINR of about 10 dB can be reached.

The average WCSINR alone does not provide much information on the quality of the individual links. We define a function N that takes the WCSINR as argument and delivers the number of links that provide at least this given WCSINR. The Figures 4(a) and 4(b) show the graph of N for each of the four discussed scenarios from Table IV for the two different placement types, respectively. For Type II placement, we observe that more than 150 links have a WCSINR of more than 10 dB if at least d_A or d_B is variable – while there are less than 50 of such links for Type I placement. In total, the improvement with respect to the WCSINR in case of Type II is clearly represented in the particular graphs of N. Regarding the best parameter setting for Type II, there are 234 links having a WCSINR of more than 5 dB, but there are six links with a WCSINR of less than -7 dB. It turns out that these are the links with the longest possible distance.

If other SNR values in a range of 5–20 dB are assumed, the optimization also yields significant improvements with respect to the achieved WCSINR and the resulting optimized antenna and board distances are very similar, see Figure 5 for an SNR of 10 dB.

We want to emphasize that in general better SINR values for the desired links can be expected since the worst-case scenario might not occur. If a large homogeneity with respect to the quality of links is of interest, then the maximization of the minimal WCSINR among (a subset of) all links might be an adequate objective function.

V. Conclusions

Using cascaded Butler matrices for wireless chip-to-chip communication yields different qualities of the links with respect to the SINR. In this paper, the influence of the array placement, the board distance, and the antenna distance on the WCSINR of wireless links has been investigated. Due to the properties of the beampatterns provided by the cascaded Butler matrices, a certain displacement between Tx and Rx arrays on the parallel boards (as for Type II placement) turned out to be beneficial. From the simulation results we observed that a low board distance (e.g. 2 cm) together with a standard antenna spacing of $d_A = 0.5\lambda$ would not yield sufficient quality of links. This can be compensated by increasing the board distance and/or the antenna element distance. In summary, the performance of such a wireless communication system can be considerably increased by optimizing these (and further) parameters individually or in combination.

ACKNOWLEDGEMENTS

This work is supported in part by the German Research Foundation (DFG) within the Collaborative Research Center 912 "Highly Adaptive Energy-Efficient Computing" (HAEC).

REFERENCES

[1] SFB 912, "Highly Adaptive Energy-Efficient Computing," http://tu-dresden.de/sfb912.

[2] A. B. Gershman, N. D. Sidiropoulos, S. Shahbazpanahi, M. Bengtsson, and B. Ottersten, "Convex optimization-based beamforming," *IEEE Signal Processing Magazine*, vol. 27, no. 3, pp. 62–75, 2010.

[3] J. Israel, A. Fischer, J. Martinovic, E. Jorswieck, and M. Mesyagutov, "Discrete receive beamforming," *IEEE Signal Processing Letters*, vol. 22, no. 7, pp. 958–962, 2015.

[4] J. Israel, A. Fischer, and J. Martinovic, "Discrete and phase-only receive beamforming," in *Modern Mathematical Methods and High Performance Computing in Science and Technology*, ser. Springer Proceedings in Mathematics & Statistics, V. K. Singh, H. M. Srivastava, E. Venturio, M. Resch, and V. Gupta, Eds. Springer, 2016, pp. 141–153.

[5] ——, "A branch-and-bound algorithm for discrete receive beamforming with improved bounds," in *2015 IEEE International Conference on Ubiquitous Wireless Broadband (ICUWB)*, Montreal, Canada, Oct. 2015.

[6] J. Butler and R. Lowe, "Beamforming matrix simplifies design of electronically scanned antennas," *Electronic Design*, vol. 9, pp. 170–173, 1961.

[7] J. Israel, J. Martinovic, A. Fischer, M. Jenning, and L. Landau, "Optimal antenna positioning for wireless board-to-board communication using a Butler matrix beamforming network," in *WSA 2013; 17th International ITG Workshop on Smart Antennas*, Stuttgart, Germany, March 2013.

[8] D. Bai, S. S. Ghassemzadeh, R. R. Miller, and V. Tarokh, "Beam selection gain from Butler matrices," in *2008 IEEE 68th Vehicular Technology Conference*, Calgary, Canada, Sept. 2008.

[9] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*. Cambridge University Press, 2005.

[10] M. Jenning, B. Klein, R. Hahnel, D. Plettemeier, D. Fritsche, G. Tretter, C. Carta, F. Ellinger, T. Nardmann, M. Schroter, K. Nieweglowski, K. Bock, J. Israel, A. Fischer, N. U. Hassan, L. Landau, M. Dörping-haus, and G. Fettweis, "Energy-efficient transceivers for ultra-highspeed computer board-to-board communication," in *2015 IEEE International Conference on Ubiquitous Wireless Broadband (ICUWB)*, Montreal, Canada, Oct. 2015.