Biological Hydrodynamics

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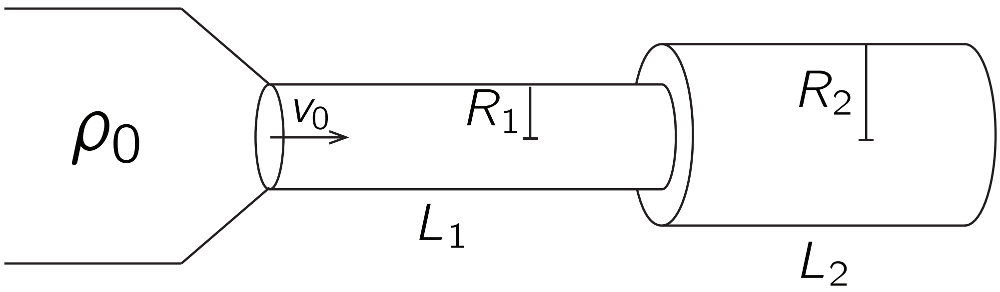
Next Tutorial: Thursday 24th October, 14:50 - 16:20, MPI PKS Seminar Room 3

Tutorial 1: Particles flowing through a pipe



Consider particles that are embedded in an incompressible solvent, and that flow with the solvent through two connected pipes with different radii R_1 and R_2, as shown in the figure. To the left, the pipes are connected to a solvent and particle reservoir. The density of particles in the reservoir is given by ρ_0. The mean flow velocity at the entrance is v_0.

1. What is the number of particles per unit time entering the first pipe?

2. Calculate the mean flow velocity and the density of particles at the exit of the second pipe.

3. Now consider a single pipe of length L and radius R that is connected to this reservoir. x denotes the position along the pipe, $0 \leq x \leq L$. $j = \rho\, v$ identifies the flux of particles. By considering the conservation of matter between two planes along the tube, show that the density of particles satisfies a continuity equation

$$\partial_t \rho + \partial_x j = 0\,. \tag{1}$$

4. We now assume that the surface of this pipe absorbs the particles embedded in the solvent, without allowing the fluid to pass through the pipe walls. Within each interval $\{x, x + \Delta x\}$, the walls absorb $2\pi f \rho R \, \Delta x$ particles per unit time, with f the surface absorption constant. Calculate the profile of average particle density $\rho(x)$ along the pipe at steady state.

5. Now we consider the same problem, but with a tube where the radius $R(x)$ decreases in a linear fashion from R_1 to R_2 over the length of the tube L ($R_1 > R_2$). Calculate the resultant particle density and velocity profiles.